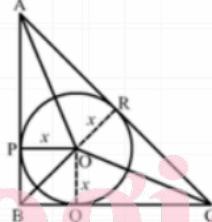


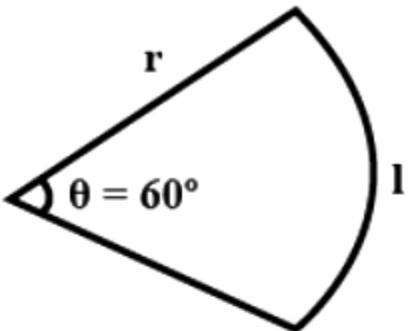
SUBJECT: MATHS

CLASS: X

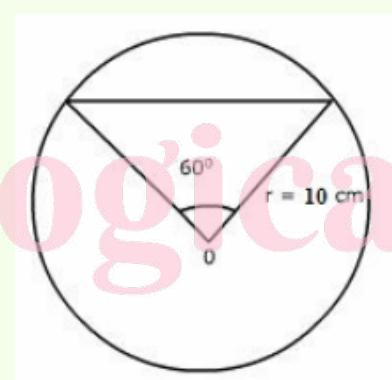
CHAPTER: Mensuration and Probability

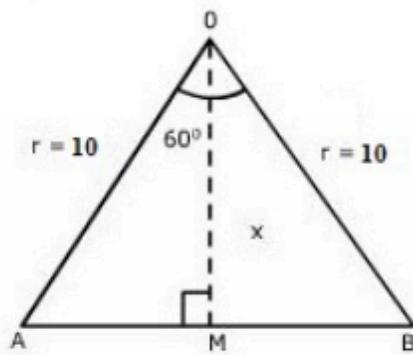
No. of PYQs:20

SI No	QUESTIONS	MARK
1	<p>In a right triangle ABC, right-angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is</p> <p>Solution-</p>  <p>In right triangle ABC</p> <p>By using Pythagoras theorem we have</p> $ \begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \\ \therefore AC^2 &= 169 \\ \Rightarrow AC &= 13 \text{ cm} \end{aligned} $ <p>Now,</p> $ \begin{aligned} Ar(\Delta ABC) &= Ar(\Delta AOB) + Ar(\Delta BOC) + Ar(\Delta AOC) \\ \Rightarrow \frac{1}{2} \times AB \times BC &= \frac{1}{2} \times OP \times AB + \frac{1}{2} \times OQ \times BC + \frac{1}{2} \times OR \times AC \\ \Rightarrow 5 \times 12 &= x \times 5 + x \times 12 + x \times 13 \\ \Rightarrow 60 &= 30x \\ \Rightarrow x &= 2 \text{ cm} \end{aligned} $	1

	[CBSE 2014]	
2	<p>A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its center. Find the radius of the circle.</p>  <p>Length, $l = 22\text{cm}$</p> <p>$\theta = 60^\circ$</p> <p>$180^\circ = \pi\text{rad}$</p> $1^\circ = \frac{\pi}{180}\text{rad}$ $60^\circ = \frac{\pi}{180} \times 60\text{ rad}$ $= \frac{\pi}{3}\text{ rad}$ <p>Arc length = $r\theta$</p> $r = \frac{1}{\theta}$ $= \frac{22}{\left(\frac{\pi}{3}\right)}$ $= \frac{(22 \times 3)}{\left(\frac{22}{7}\right)}$ $r = 3 \times 7$ $r = 21\text{ cm}$	2

[CBSE 2020]

3	<p>A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle 120°. Find the total area cleaned at each sweep of the blades.</p> <p>Total area cleaned = $2 \times$ Area of sector</p> $= 2 \times \frac{\pi r^2 \theta}{360^\circ}$ $= 2 \times \frac{22}{7} \times 21 \times 21 \times \frac{120^\circ}{360^\circ}$ $= 924 \text{ cm}^2$ <p>Hence the total area cleaned at each sweep of the blades is 924 cm^2.</p> <p style="text-align: right;">[CBSE2019]</p>	3
4	<p>A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the center of the circle. Find the area of major and minor segments of the circle.</p>  <p>Area of sector formed by the arc = $\frac{\theta}{360^\circ} \times \pi r^2$</p> $= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 10^2$ $= \frac{1100}{21}$ $= 52.38 \text{ cm}^2$	5



In $\triangle OAB$, let OM be \perp bisector of AB ,

$$\angle AOM = \angle BOM$$

Let OM be x cm

In $\triangle OMA$,

$$\frac{OM}{OA} = \cos 30^\circ$$

$$\frac{x}{10} = \frac{\sqrt{3}}{2}$$

$$x = 5\sqrt{3}$$

$$= 8.66 \text{ cm}$$

$$OM = 8.66 \text{ cm}$$

In $\triangle OMA$,

$$\frac{AM}{OA} = \sin 30^\circ$$

$$\frac{AM}{10} = \frac{1}{2}$$

$$AM = 5 \text{ cm}$$

$$AB = 2 \times AM = 2 \times 5 = 10 \text{ cm}$$

$$\text{Area of } \triangle OAB = \frac{1}{2} \times OM \times AB$$

$$= \frac{1}{2} \times 8.66 \times 10$$

$$= 43.3 \text{ cm}^2$$

$$\text{Area of minor segment} = \text{Area of sector formed by the arc} - \text{Area of } \triangle OAB$$

$$= 52.38 - 43.3$$

$$= 9.08 \text{ cm}^2$$

$$\text{Area of major segment} = \text{Area of circle} - \text{Area of minor segment}$$

$$= \pi r^2 - 9.08$$

$$= 3.14 \times 10 \times 10 - 9.08$$

$$= 314 - 9.08$$

$$= 304.92 \text{ cm}^2$$

[CBSE 2017]

5	<p>In a circle of diameter 42 cm, if an arc subtends an angle of 60° at the center where $\pi = 22/7$, then what will be the length of arc?</p> $\text{Length of Arc APB} = \frac{\theta}{360^\circ} \times (2\pi r) \quad \left. \right\} \frac{1}{2} \text{ marks}$ $= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$ $= \frac{1}{6} \times 2 \times \frac{22}{7} \times 21$ $= 22 \text{ cm} \quad \left. \right\} \frac{1}{2} \text{ marks}$	1
6	<p>In the given figure, arcs have been drawn of radius 7 cm each with vertices A, B, C and D of quadrilateral ABCD as centers. Find the area of the shaded region</p>	1
7	<p>The curved surface area of a cone having height 24 cm and radius 7 cm, is</p>	1

Given, Radius of cone, $r = 7\text{cm}$ and Height, $h = 24\text{cm}$.

To find slant height l :

We know, $l^2 = r^2 + h^2$

$$\Rightarrow l^2 = 7^2 + 24^2$$

$$\Rightarrow l^2 = 625$$

$$\Rightarrow l = 25\text{cm.}$$

Therefore, curved surface area of cone is $= \pi r l$

$$= \frac{22}{7} \times 7 \times 25$$

$$= 22 \times 25$$

$$= 550\text{cm}^2.$$

[CBSE 2023]

8

A solid spherical ball fits exactly inside the cubical box of side a . The volume of the ball is

Given the solid ball is exactly fitted into cubical box of side a

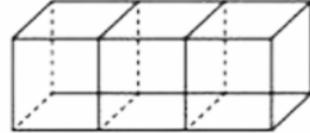
Therefore the radius of the solid ball is exactly half of the side of the cubical box

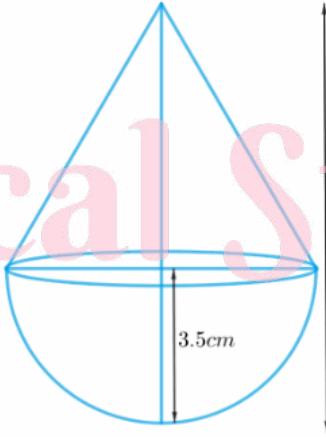
$$\therefore r = \frac{a}{2}$$

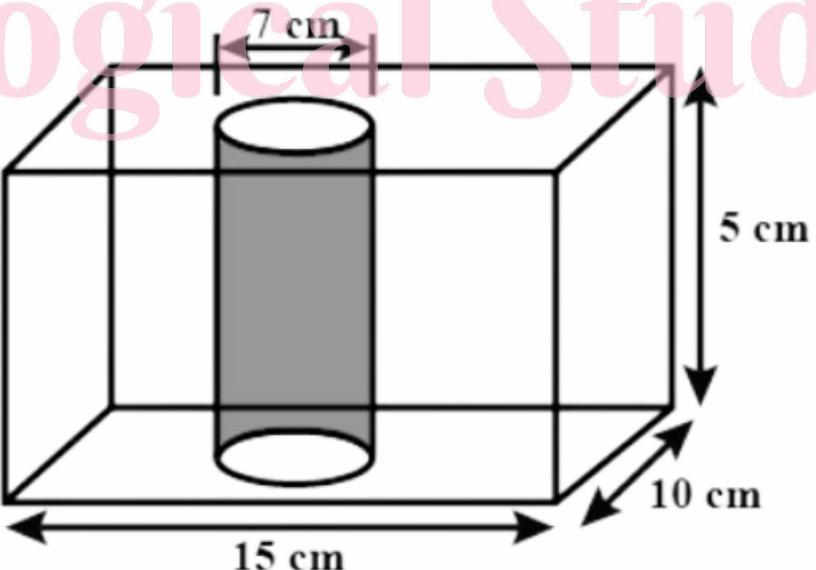
$$\text{Volume of solid ball} = \frac{4}{3} \cdot \pi \cdot r^3 = \frac{4}{3} \cdot \pi \cdot \frac{a^3}{8}$$

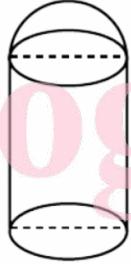
$$\therefore \text{Volume of solid ball} = \frac{\pi a^3}{6}$$

[CBSE 2020]

9	<p>The radius of the base and the height of a solid right circular cylinder are in the ratio 2:3 and its volume is 1617 cm^3. Find the total surface area of the cylinder.</p> <p>Let $r = 2x$ and $h = 3x$.</p> $V = \pi r^2 h$ $\Rightarrow \frac{22}{7} \times (4x^2)(3x) = 1617$ $\Rightarrow x^3 = 42.875$ $\Rightarrow x = 3.5 \text{ cm}$ <p>Hence, $r = 7 \text{ cm}$ and $h = 10.5 \text{ cm}$</p> <p>Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$</p> $= 2 \times \frac{22}{7} \times 7(17.5)$ $= 770 \text{ cm}^2$	3
10	<p>Three cubes 6 cm edge each, are joined as shown in the given figure. Find the total surface area of the resulting cuboid.</p> 	3

	<p>Edge of each cube = 6 cm If 3 cubes are joined end to end, then Length of newly formed cuboid = $3 \times 6 = 18$ cm Breadth of newly formed cuboid = 6 cm Height of the newly formed cuboid = 6 cm As we know, Total surface area of cuboid = $2(lb + bh + hl) = 2(18 \times 6 + 6 \times 6 + 6 \times 18) = 2(108 + 36 + 108) = 2 \times 252 = 504$ cm²</p> <p style="text-align: right;">[CBSE 2021]</p>	
11	<p>A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy.</p> <p>We can create the figure of the toy as per the given information</p>  <p>CSA of the cone = $\pi r l$, where 'r' and 'l' are the radius and slant height of the cone respectively.</p> <p>Radius of the hemisphere, $r = 3.5$ cm</p> <p>Height of the hemisphere = radius of the hemisphere, $r = 3.5$ cm</p> <p>Radius of the cone, $r = 3.5$ cm</p> <p>Height of the cone = Total height of the toy - height of the hemisphere</p> <p>$h = 15.5$ cm - 3.5 cm = 12 cm</p> <p>Slant height of the cone, $l = \sqrt{r^2 + h^2}$</p> <p>$l = \sqrt{(3.5 \text{ cm})^2 + (12 \text{ cm})^2}$</p> <p>$l = \sqrt{12.25 \text{ cm}^2 + 144 \text{ cm}^2}$</p>	3

	$l = \sqrt{56.25 \text{ cm}^2}$ $l = 12.5 \text{ cm}$ <p>Total surface area of the toy = CSA of the hemisphere + CSA of the cone</p> $= 2\pi r^2 + \pi r l$ $= \pi r (2r + l)$ $= 22/7 \times 3.5 \text{ cm} \times (2 \times 3.5 \text{ cm} + 12.5 \text{ cm})$ $= 22/7 \times 3.5 \text{ cm} \times (7 \text{ cm} + 12.5 \text{ cm})$ $= 11 \text{ cm} \times 19.5 \text{ cm}$ $= 214.5 \text{ cm}^2$ <p style="text-align: right;">[CBSE 2015, Sample Paper 2016]</p>	
12	<p>In the given figure, from a cuboidal solid metallic block, of dimensions $15 \text{ cm} \times 10 \text{ cm} \times 5 \text{ cm}$, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block.</p> 	3

	<p>Surface area of remaining block = Surface area of cuboid + Curved surface area of cylinder - $2 \times$ area of base</p> $= A_I + A_{II}$ $A_{II} = 2\pi(3.5)5 - 2\pi(3.5)^2 = 33\text{cm}^2$ $A_I = 2(15 \times 10 + 10 \times 5 + 5 \times 15) = 2 \times 275 = 550\text{cm}^2$ $A_{\text{total}} = 583\text{cm}^2$	
13	<p>[CBSE 2015]</p> <p>A room is in the form of a cylinder surmounted by a hemi-spherical dome. The base radius of the hemisphere is $\frac{2}{3}rd$ the height of the cylindrical part. Find total height of the room if it contains $67\frac{1}{21}$ cube meter of air.</p> 	3

	<p>Height of building = height of cylinder + height of hemisphere $H = h + r$ r is radius of cylinder and dome diameter of dome = $2/3 \times$ height of building $\therefore 2r = \frac{2}{3} \times (h + r)$ $\therefore 6r = 2h + 2r \therefore 3r = h + r$ $\therefore h = 2r$ Volume of air in building = Volume of cylinder + Volume of dome $\therefore 67 \frac{1}{21} = \pi r^2 h + \frac{2}{3} \pi r^3$ $\therefore \frac{1408}{21} = \pi r^2 (h + \frac{2}{3}r)$ $= \pi r^2 (2r + 2/3r)$ $= \pi r^2 (\frac{8}{3}r)$ $\therefore \frac{1408}{21} = \frac{8}{3} \pi r^3$ $\therefore \frac{1408}{7} = 8 \times \frac{22}{7} \times r^3$ $\therefore r^3 = \frac{1408}{22 \times 8} = 8$ $\therefore r = 2 \text{ m} \Rightarrow h = 4 \text{ m}$ Height of building = $r + h = 6 \text{ m}$ </p>	
14	<p>The largest possible sphere is carved out of a wooden solid cube of 7 cm side. Find the volume of the wood left.</p> <p>Given that, a largest possible sphere is carved out of a wooden solid cube of side 7 cm. Diameter of sphere, $d = 7 \text{ cm}$. Radius of sphere = $\frac{\text{diameter}}{2}$ $\Rightarrow r = \frac{7}{2} \text{ cm}$ Volume of the sphere, $= \frac{4}{3} \pi r^3$. $[\pi = \frac{22}{7}]$ \Rightarrow Volume of the sphere, $= \frac{4}{3} \times \frac{22}{7} \times (\frac{7}{2})^3$ \Rightarrow Volume of the sphere $= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \Rightarrow$ Volume of the sphere $= 179.67 \text{ cm}^3$ Volume of a cube $= a^3$ $\Rightarrow (7)^3 \Rightarrow 7 \times 7 \times 7 \Rightarrow 343 \text{ cm}^3$ Volume of wood left = Volume of a cube - Volume of the sphere. Volume of wood left $= 343 - 179.67$ Volume of wood left $= 163.33 \text{ cm}^3$.</p> <p style="text-align: right;">[CBSE 2015, 2016]</p>	3
15	<p>The curved surface area of a right circular cone is 12320 cm^2. If the radius of its base is 56 cm, then find its height.</p>	2

radius of the base of the cone = 56cm

We know that curved surface area of a cone = $\pi r l$.

Given that curved surface area of a cone = 12320.

$$\pi r l = 12320$$

$$\frac{22}{7} * 56 * l = 12320$$

$$22 * 8 * l = 12320$$

$$176 * l = 12320$$

$$l = \frac{12320}{176} = 70.$$

We know that height of the cone,

$$h = \sqrt{l^2 - r^2}$$

$$= \sqrt{70^2 - 56^2}$$

$$= \sqrt{4900 - 3136}$$

$$= \sqrt{1764}$$

$$= 42.$$

Therefore the height of the cone = 42m.

[CBSE 2021-22]

16	<p>In a group of 20 people, 5 can't swim. If one person is selected at random, then the probability that he/she can swim, is</p> <p>Solution- Persons who can swim= 20-5=15</p> <p>Probability who can swim= $\frac{15}{20} = \frac{3}{4}$</p> <p style="text-align: right;">[CBSE 2021, 2017]</p>	1
17	<p>Two dice are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is</p> <p>When two dice are thrown,</p> <p>The sample space (S) =</p> $\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$ <p>$\therefore n(S) = 36$</p> <p>Let A: Event of getting the numbers whose difference is 3.</p> <p>Let A: Event of getting the numbers whose difference is 3.</p> <p>$\therefore A = \{(1, 4), (2, 5), (3, 6), (4, 1), (5, 2), (6, 3)\}$</p> <p>$n(A) = 6$</p> <p>$\therefore \text{Required probability} = \frac{n(A)}{n(S)}$</p> $= \frac{6}{36}$ $= \frac{1}{6}$ <p style="text-align: right;">[CBSE 2017]</p>	1
18	If two different dice are rolled together, the probability of getting an even number on both dice, is	1

	<p>Solve for the favorable outcomes and find the required probability:</p> <p>Possible outcomes of rolling the two dice are:</p> <p>$\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$</p> <p>Total number of outcomes = 36</p> <p>The favorable outcomes are:</p> <p>$\{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$.</p> <p>Total number of favorable outcomes = 9</p> <p>Probability of getting an even number on both dice = $\frac{9}{36} = \frac{1}{4}$</p> <p>Hence, the probability of getting an even number on both dice is $\frac{1}{4}$</p> <p style="text-align: right;">[CBSE 2021, 2014]</p>	
19	<p>A bag contains 4 red, 3 blue and 2 yellow balls. One ball is drawn at random from the bag. Find the probability that drawn ball is (i) red (ii) yellow</p> <p>Solution- $P(\text{red}) = \frac{4}{9}$</p> <p>$P(\text{yellow}) = \frac{2}{9}$</p> <p style="text-align: right;">[CBSE 2015, 2020]</p>	3
20	<p>Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the number 25.</p> <p>Solution-</p>	5

Let us first write the all possible outcomes when Peter throws two different dice together.

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

∴ Total number of outcomes = 36

The favorable outcome for getting the product of numbers on the dice equal to 25 is (5, 5).
Favourable number of outcomes = 1

∴ Probability that Peter gets the product of numbers as 25

$$= \frac{\text{Favourable number of outcomes}}{\text{Total number of outcomes}} = \frac{1}{36}$$

The outcomes when Rina throws a die are 1, 2, 3, 4, 5, 6

∴ The favorable outcome for getting the product of numbers on the dice equal to 25 is (5, 5).
Favourable number of outcomes = 1

∴ Probability that Peter gets the product of numbers as 25

$$= \frac{\text{Favourable number of outcomes}}{\text{Total number of outcomes}} = \frac{1}{36}$$

The outcomes when Rina throws a die are 1, 2, 3, 4, 5, 6

∴ Total number of outcomes = 6

Rina throws a die and squares the number, so to get the number 25, the faces should be 5.

Favourable number of outcomes = 1

Favourable number of outcomes = 1

∴ Probability that Rina gets the square of the number as 25

$$= \frac{\text{Favourable number of outcomes}}{\text{Total number of outcomes}} = \frac{1}{6}$$

As, $1/6 > 1/36$, so Rina has better chance to get the number 25.

[CBSE 2015]

Logical Study

Logical Study

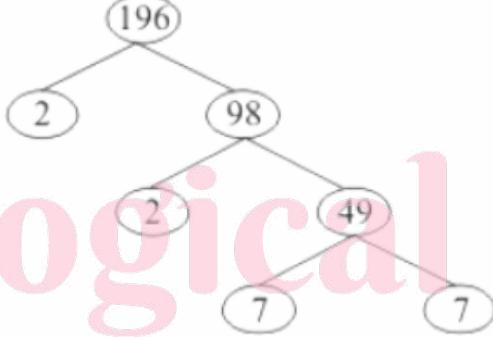
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SUBJECT: Maths

CLASS: X

CHAPTER: Algebra & Real numbers

No. of PYQs:20

SI No	QUESTIONS	MARK
1	<p>The sum of the exponents of the prime factors in the prime factorisation of 196, is</p> <p>Using the factor tree for prime factorization, we have:</p>  <p>Therefore,</p> $196 = 2 \times 2 \times 7 \times 7$ $196 = 2^2 \times 7^2$ <p>The exponents of 2 and 7 are 2 and 2 respectively.</p> <p>Thus the sum of the exponents is 4</p> <p style="text-align: right;">[CBSE 2018]</p>	1
2	An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?	2

	<p>An army contingent of 612 members is to march behind an army band of 48 members. Here, HCF of 612 and 48 will give the maximum number of columns in which the two groups can march.</p> <p>So, using Euclid's division algorithm</p> $612 = 48 \times 12 + 36$ $\Rightarrow 48 = 36 \times 1 + 12$ $\rightarrow 36 = 12 \times 3 + 0$ $\therefore \text{HCF}(612, 48) = 12$ <p>Hence, the maximum no of columns in which they can march is 12</p>	
3	<p>If HCF of 65 and 117 is expressible in the form $65n - 117$, then find the value of n.</p> $65 = 5 \times 13$ $117 = 3 \times 3 \times 13$ <p>Therefore, HCF of 65 and 117 is 13.</p> $\text{So, } 65m - 117 = 13$ $\text{or, } 65m = 130$ <p>Therefore, $m = 2$</p>	2
4	<p>If one zero of $3x^2 + 8x + k$ be the reciprocal of the other then $k =$</p> <p>Let x and $\frac{1}{x}$ be the zeroes of the polynomial $3x^2 + 8x + k$.</p> <p>Product of the polynomial = $x \cdot \frac{1}{x} = \frac{k}{3}$</p> $\Rightarrow 1 = \frac{k}{3}$ $\Rightarrow k = 3$ <p>Thus, $k = 3$.</p>	2
5	<p>Find the zeros of $2\sqrt{3}x^2 - 5x + \sqrt{3}$ and verify the relationship between the zeros and the coefficients</p>	

$$\text{Let } f(x) = 2\sqrt{3}x^2 - 5x + \sqrt{3}$$

$$= 2\sqrt{3}x^2 - 2x - 3x + \sqrt{3}$$

$$= 2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1)$$

to find the zeroes, Let $f(x) = 0$

$$(\sqrt{3}x - 1) \text{ or } (2x - \sqrt{3}) = 0$$

$$x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \text{ or } x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\sqrt{3}}{3} \text{ or } x = \frac{\sqrt{3}}{2}$$

Again,

Sum of zeroes =

$$\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{6} = \frac{5}{2\sqrt{3}} = \frac{-b}{a} = \frac{(-\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{3}{6} = \frac{1}{2} = \frac{c}{a} = \frac{\text{Constant term}}{\text{coefficient of } x^2}$$

[CBSE 2011,2020]

6

If α and β are zeroes of the quadratic polynomial $4x^2 + 4x + 1$, then form a quadratic polynomial whose zeroes are 2α and 2β

2

to solve this let's divide the quadratic equation by 4 such that we have it in the form $x^2 + (\alpha + \beta)x + \alpha\beta$

so after dividing by 4 we have

$$\begin{array}{r} 4x^2 + 4x + 1 \\ \hline 4 \\ x^2 + x + \frac{1}{4} \end{array}$$

So we have $\alpha + \beta = 1$ (1)

and $\alpha\beta = \frac{1}{4}$ (2)

So if zeroes to the new quadratic equation are 2α and 2β , then

$$\begin{aligned} 2\alpha + 2\beta &= 2(\alpha + \beta) \\ &= 2(1) = 2 \quad \text{Using (1)} \end{aligned}$$

and,

$$\begin{aligned} 2\alpha \times 2\beta &= 4\alpha\beta \\ &= 4 \times \frac{1}{4} = 1 \quad \text{Using (2)} \end{aligned}$$

So the new quadratic equation with its roots 2α and 2β will be

$$x^2 + (2\alpha + 2\beta)x + 2\alpha \times 2\beta$$

Putting the values, the equation would be,

$$x^2 + 2x + 1$$

[CBSE 2015]

7

The value of k for which the equations $3x - y + 8 = 0$ and $6x + ky = -16$ represent coincident lines, is:

1

Condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \dots (i)$$

Given lines:

$$3x - y + 8 = 0$$

$$6x - ky + 16 = 0$$

Here,

$$a_1 = 3, b_1 = -1, c_1 = 8$$

$$a_2 = 6, b_2 = -k, c_2 = 16$$

from Eq. (i)

$$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$$

$$\Rightarrow \frac{1}{k} = \frac{1}{2}$$

$$\therefore k = 2$$

[CBSE 2018, 2013]

8

Solve: $ax + by = a - b$ & $bx - ay = a + b$.

2

	$ax + by = a - b \quad \dots\dots(1)$ $bx - ay = a + b \quad \dots\dots(2)$ <p>Now we multiply both of the equation,</p> $\Rightarrow a^2x + aby = a^2 - ab$ $\Rightarrow b^2x - aby = ab + b^2$ $\Rightarrow (a^2 + b^2)x = a^2 + b^2$ $\Rightarrow x = (a^2 + b^2)/(a^2 + b^2) = 1$ <p>Now we will put the value x in equation (1)</p> $\Rightarrow a(1) + by = a - b$ $\Rightarrow by = a - b - a$ $\Rightarrow y = -1$ <p style="text-align: right;">[CBSE 2018,2012,]</p>	
9	Solve the system of equations graphically: $3x - 4y = 7$ and $5x + 2y = 3$ Shade the region between the lines and the y-axis	2

Now,

$$\begin{aligned}3x - 4y &= 7 \\ \Rightarrow 3x - 7 &= 4y \\ \Rightarrow 4y &= 3x - 7 \\ \Rightarrow y &= \frac{3x - 7}{4}\end{aligned}$$

When $x = 1$, we have

$$y = \frac{3 \times 1 - 7}{4} = -1$$

When $x = -3$, we have

$$y = \frac{3 \times (-3) - 7}{4} = -4$$

Thus, we have the following table:

x	1	-3
y	-1	-4

We have,

$$\begin{aligned}5x + 2y &= 3 \\ \Rightarrow 2y &= 3 - 5x \\ \Rightarrow y &= \frac{3 - 5x}{2}\end{aligned}$$

When $x = 1$, we have

$$y = \frac{3 - 5 \times 1}{2} = -1$$

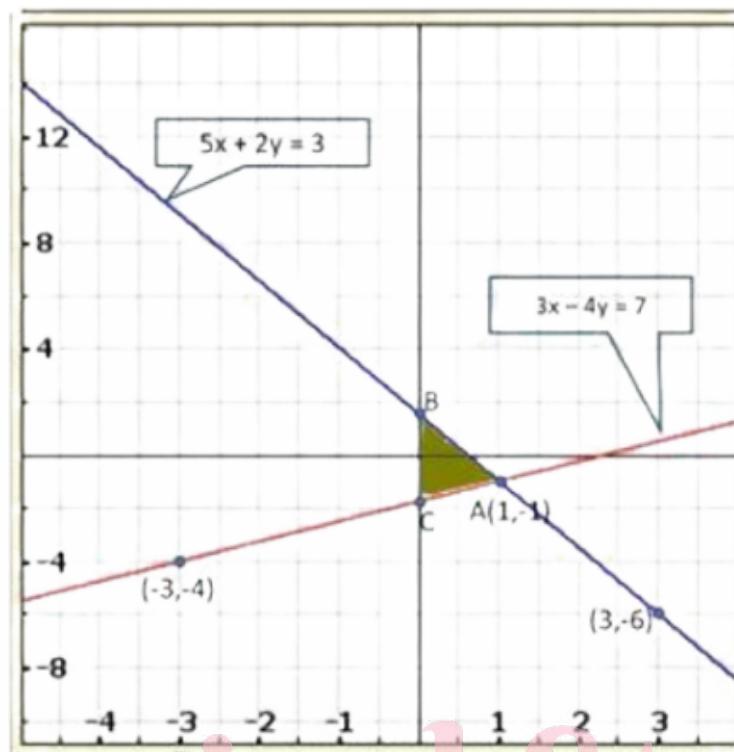
When $x = 3$, we have

$$y = \frac{3 - 5 \times 3}{2} = -6$$

Thus, we have the following table:

x	1	3
y	-1	-6

Graph of the given system of equations:



[CBSE 2006, 2002]

10	Solve the following pair of linear equations for x and y : $2(ax - by) + (a + 4b) = 0$; $2(bx + ay) + (b - 4a) = 0$	2
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	<p>The given equations may be written as</p> $2ax - 2by = -a - 4b \quad \dots \text{(i)}$ $2bx + 2ay = 4a - b \quad \dots \text{(ii)}$ <p>Multiplying (i) by a and (ii) by b and adding , we get</p> $(2a^2 + 2b^2)x = (-a^2 - b^2)$ $\Rightarrow 2(a^2 + b^2)x = -(a^2 + b^2) \Rightarrow x = \frac{-1}{2}$ <p>Putting $x = \frac{-1}{2}$ in (i), we get</p> $2a \times \left(\frac{-1}{2}\right) - 2by = -a - 4b$ $\Rightarrow -a - 2by = -a - 4b$ $\Rightarrow 2by = 4b \Rightarrow y = \frac{4b}{2b} = 2$ <p>Hence, $x = \frac{1}{2}$ and $y = 2$</p> <p style="text-align: right;">[CBSE 2012,2014]</p>	
11	The discriminant of the quadratic equation $4x^2 - 6x + 3 = 0$	2

	<p>Since we have given that</p> $4x^2 - 6x + 3 = 0$ <p>Here, $a = 4$</p> <p>$b = -6$</p> <p>$c = 3$</p> <p>So, the value of discriminant would be</p> $D = \sqrt{b^2 - 4ac}$ $D = \sqrt{(-6)^2 - 4 \times 4 \times 3}$ $D = \sqrt{36 - 48}$ $D = \sqrt{-12}$ $D = 2\sqrt{3}i$ <p style="text-align: right;">[CBSE 2012,2014]</p>	
12	If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$	2

	<p>number we have $10(6) + 1(3) = 60 + 3 = 63$ So the number is 63</p> <p style="text-align: right;">[CBSE 2016, 2014]</p>	
14	<p>If a student had walked 1 km/hr faster, he would have taken 15 minutes less to walk 3 km. Find the rate at which he was walking.</p> <p>Let normal speed of a student be $x \text{ km/h}$ Distance (d) = 3 km Time = distance/speed = $3/x$ hours If a student had walk 1 km/h faster then his speed would have been $x + 1 \text{ km/h}$. Time = $3/(x+1)$ $\frac{3}{x} - \frac{3}{x+1} = \frac{15}{60}$ $3\left(\frac{1}{x} - \frac{1}{x+1}\right) = \frac{1}{4}$ $12\left(\frac{x+1-x}{x(x+1)}\right) = 1$ $12 = x^2 + x$ $x^2 + x - 12 = 0$ $(x + 4)(x - 3) = 0$ $x = -4 \text{ or } x = 3$ $x \neq -4 \text{ hence } x = 3 \text{ km/hour}$</p> <p style="text-align: right;">[CBSE 2015, 2018]</p>	2
15	The 5th term of an AP is 20 and the sum of its 7th and 11th terms is 64. The common difference of the AP is	3

	<p>Let the first term of AP be a and common difference be d.</p> <p>According to question,</p> <p>5th term is $20 \Rightarrow a + 4d = 20 \dots\dots(1)$</p> <p>And, $a + 6d + a + 10d = 64$</p> <p>$\Rightarrow 2a + 16d = 64$</p> <p>$\Rightarrow a + 8d = 32 \dots\dots(2)$</p> <p>From (1) & (2),</p> <p>$d = 3, \text{ & } a = 8$</p> <p>Hence common Difference = 3</p> <p style="text-align: right;">[CBSE 2015,2013]</p>	
16	<p>Find the sum of all 11 terms of an A.P. whose middle most term is 30</p> <p>Number of terms are 11, so $n = 11$</p> <p>Middle term = $\frac{11 + 1}{2} = \frac{12}{2} = 6th$ term</p> <p>Also, middle term = 30 [Given]</p> <p>$\therefore a_6 = 30$ [Given]</p> <p>$\Rightarrow a + (6 - 1)d = 30$</p> <p>$\Rightarrow a + 5d = 30 \dots(i)$</p> <p>$\because S_n = \frac{n}{2} [2a + (n - 1)d]$</p> <p>$\Rightarrow S_{11} = \frac{11}{2} [2a + (11 - 1)d]$</p>	2

	$= \frac{11}{2} [2a + 10d]$ $= \frac{11 \times 2}{2} [a + 5d]$ $= 11 \times 30 \text{ [Using (i)]}$ $\Rightarrow S_{11} = 330$ <p>Hence, the sum of all 11 terms is 330.</p>	
	[CBSE 2015]	
17	<p>Find the common difference of an AP whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.</p> <p>Let d is common difference of AP</p> <p>Now first 4 terms are $5, 5 + d, 5 + 2d, 5 + 3d$ and next 4 terms $5 + 4d, 5 + 5d, 5 + 6d, 5 + 7d$</p> <p>Given that, the sum of its first four terms is half the sum of the next four terms. i.e.,</p> $5 + 5 + d + 5 + 2d + 5 + 3d = \frac{5 + 4d + 5 + 5d + 5 + 6d + 5 + 7d}{2}$ $20 + 6d = \frac{(20 + 22d)}{2}$ $20 + 6d = 10 + 11d$ $d = 2$ <p>Hence, the common difference of the given A.P. is 2</p>	3
	[CBSE 2012,2018]	
18	Solve the equation for x : $1 + 4 + 7 + 10 + \dots + x = 287$.	2

Here, given

$$a = 1$$

$$d = 4 - 1 = 3$$

$$\text{and, } s_n = 287$$

Now,

$$s_n = \frac{n}{2}(2a + (n - 1)d)$$

$$\Rightarrow 287 = \frac{n}{2}(2 \times 1 + (n - 1)3)$$

$$\Rightarrow 287 = \frac{n}{2}(2 + 3n - 3)$$

$$\Rightarrow 574 = n(3n - 1)$$

$$\Rightarrow 574 = 3n^2 - n$$

$$\Rightarrow 3n^2 - n - 574 = 0$$

on solving the quadratic equation using formula

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We get $n = 14$ & $\frac{-41}{3}$ [does not exist]

so, $n = 14$

Now,

$$s_n = \frac{n}{2}(a + 1)$$

$$\Rightarrow 287 = \frac{14}{2}(1 + x)$$

$$\Rightarrow 574 = 14(1 + x)$$

$$\Rightarrow (1 + x) = \frac{574}{14}$$

$$\Rightarrow 1 + x = 41$$

$$\Rightarrow x = 41 - 1$$

$$\therefore x = 40$$

$x = 40$ is the solution.

[CBSE 2017, 2020]

19

If the n th term of an A.P. is $pn + q$, find its common difference.

1

	<p>The first multiple of 4 that is greater than 10 is 12.</p> <p>The next multiple will be 16.</p> <p>Therefore, the series formed as;</p> <p>12, 16, 20, 24, ...</p> <p>All these are divisible by 4 and thus, all these are terms of an AP with the first term as 12 and the common difference as 4.</p> <p>When we divide 250 by 4, the remainder will be 2. Therefore, $250 - 2 = 248$ is divisible by 4.</p> <p>The series is as follows.</p> <p>12, 16, 20, 24, ..., 248</p> <p>Let 248 be the nth term of this AP.</p> <p>First term, $a = 12$</p> <p>Common difference, $d = 4$</p> <p>$a_n = 248$</p> <p>As we know,</p> $a_n = a + (n - 1) d$ $248 = 12 + (n - 1) \times 4$ $\Rightarrow 236/4 = n - 1$ $\Rightarrow 59 = n - 1$ $\Rightarrow n = 60$	
20	Find the number of terms of the A.P.: 293, 285, 277, 53	2

[CBSE 2019]

Let the numbers be $a - d, a, a + d$

$$\text{so } 3a = 27$$

$$\Rightarrow a = 9$$

$$\text{Also } (a - d)^2 + a^2 + (a + d)^2 = 293.$$

$$3a^2 + 2d^2 = 293$$

$$d^2 = 25$$

$$\Rightarrow d = \pm 5$$

therefore numbers are 4, 9, 14.

[CBSE Term II, 2021-22]

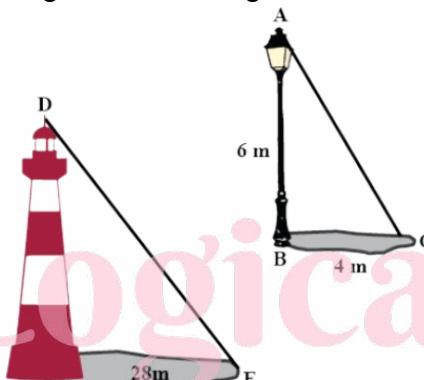
Logical Study

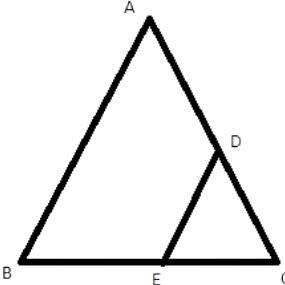
SUBJECT: Maths

CLASS: X

CHAPTER: Geometry and Co-ordinate Geometry

No. of PYQs:20

SI No	QUESTIONS	MARK
1	<p>A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.</p> 	1

	<p>Height of pole = AB = 6 m</p> <p>Length of shadow of pole = BC = 4 m</p> <p>Length of shadow of tower = EF = 28 m</p> <p>In $\triangle ABC$ and $\triangle DEF$</p> <p>$\angle B = \angle E = 90^\circ$ both 90° as both are vertical to ground</p> <p>$\angle C = \angle F$ (same elevation in both the cases as both shadows are cast at the same time)</p> <p>$\therefore \triangle ABC \sim \triangle DEF$ by AA similarity criterion</p> <p>We know that if two triangles are similar, ratio of their sides are in proportion</p> <p>So, $\frac{AB}{DE} = \frac{BC}{EF}$</p> $\Rightarrow \frac{6}{DE} = \frac{4}{28}$ $\Rightarrow DE = 6 \times 7 = 42 \text{ m}$ <p style="text-align: right;">[CBSE 2016]</p>	
2	<p>In the figure of $\triangle ABC$, the points D and E are on the sides CA, CB respectively such that $DE \parallel AB$, $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$. Then, find x..</p> 	2

In ABC , $DE \parallel AB$

$$\frac{CD}{CA} = \frac{CE}{CB}$$

or, $\frac{CD}{CD+AD} = \frac{CE}{CE+BE}$

or, $\frac{x+3}{x+3+2x} = \frac{x}{x+2x-1}$

or, $\frac{x+3}{3x+3} = \frac{x}{3x-1}$

or, $(x+3)(3x-1) = x(3x+3)$

or, $3x^2 - x + 9x - 3 = 3x^2 + 3x$

or, $8x - 3 = 3x$

or, $8x - 3x = 3$

or, $5x = 3$

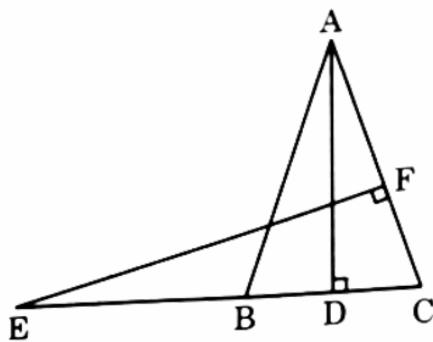
or, $x = \frac{3}{5}$

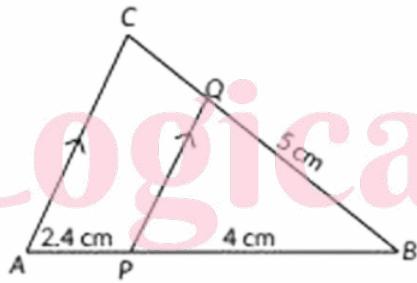
[CBSE 2016]

3

In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, then prove that $\triangle ABD \sim \triangle ECF$

2





As $PQ \parallel AC$ by using basic proportionality theorem

$$\Rightarrow \frac{BP}{PA} = \frac{BQ}{QC}$$

$$\Rightarrow \frac{4}{2.4} = \frac{5}{QC}$$

$$\Rightarrow QC = \frac{5 \times 2.4}{4} = 5 \times 0.6$$

$$\Rightarrow QC = 3 \text{ cm}$$

$$\therefore BC = BQ + QC$$

$$= 5 + 3$$

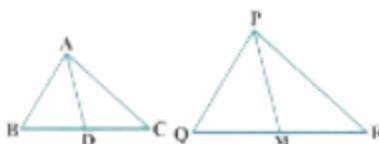
$$= 8 \text{ cm}$$

[CBSE 2023]

5

If sides AB , BC and median AD of $\triangle ABC$ are proportional to the corresponding sides PQ , QR and median PM of $\triangle PQR$, show that $\triangle ABC \sim \triangle PQR$.

2



Since AD and PM are medians of $\triangle ABC$ and $\triangle PQR$,
 $\therefore BD = \frac{1}{2}BC$ and $QM = \frac{1}{2}QR$(1)

Given that,

$$\frac{AB}{PO} = \frac{BC}{QR} = \frac{AD}{PM} \dots\dots\dots (2)$$

∴ From (1) and (2),

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \dots \dots \dots (3)$$

In $\triangle ABD$ and $\triangle PQM$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

∴ By SSS criteriam of proportionality $\triangle ABD \sim \triangle PQM$

$\therefore \angle B = \angle Q$ (Corresponding Sides of Similar Triangles) (4)

In $\triangle ABC$ and $\triangle PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} \text{ (From 2)}$$

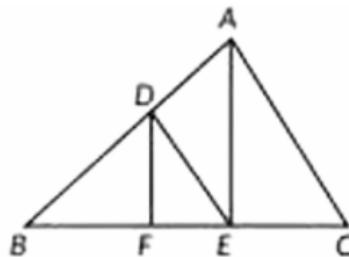
$\angle B = \angle Q$ (From 4)

∴ By SAS criterion of proportionality $\triangle ABC \sim \triangle PQR$

[CBSE 2011,2020]

6

In the given figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $BF/FE = BE/EC$



In $\triangle ABC$

$DE \parallel AC$

Line drawn parallel to one side of triangle, intersects the other two sides. It divides the other side in same ratio.

$$\frac{BE}{EC} = \frac{BD}{DA} \quad \text{(i)}$$

In $\triangle AEB$

$DF \parallel AE$

Line drawn parallel to one side of triangle, intersects the other sides. It divides the other sides in same ratio.

$$\frac{BF}{FE} = \frac{BD}{DA} \quad \text{(ii)}$$

From (i) & (ii)

$$\frac{BE}{EC} = \frac{BF}{FE}$$

\therefore Hence proved.

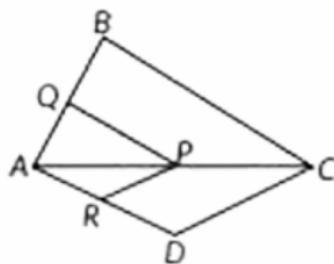
[CBSE 2020]

2

7

In figure, if $PQ \parallel BC$ and $PR \parallel CD$, prove that $QB/AQ = DR/AR$

1



(II) In $\triangle ABC$, $QB/AQ = PC/AP$ by BPT ... (1)

In $\triangle ACD$, $\frac{PC}{AP} = \frac{DR}{AR}$ by BPT. ... (2)

From (1) & (2)

$$\frac{QB}{AQ} = \frac{PC}{AP} = \frac{DR}{AR}$$

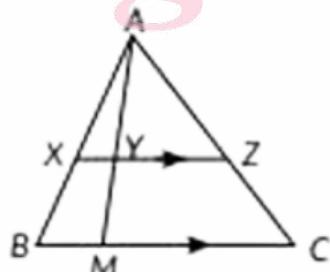
$$\therefore \frac{QB}{AQ} = \frac{DR}{AR}$$

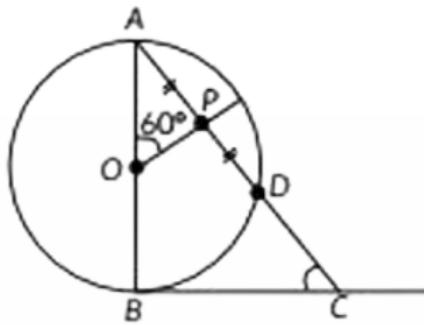
It is proved.

[CBSE 2020, 2013]

8

In the given figure, XZ is parallel to BC . $AZ = 3 \text{ cm}$, $ZC = 2 \text{ cm}$, $BM = 3 \text{ cm}$ and $MC = 5 \text{ cm}$. Find the length of XY . 2





Since, OP bisects the chord AD, therefore $\angle OPA = 90^\circ$...[\because The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]

Now, In $\triangle AOP$,

$$\begin{aligned}\angle A &= 180^\circ - 60^\circ - 90^\circ \\ &= 120^\circ - 90^\circ \\ &= 30^\circ\end{aligned}$$

Also, we know that the tangent at any point of a circle is perpendicular to the radius through the point of contact

$$\therefore \angle ABC = 90^\circ$$

Now, In $\triangle ABC$,

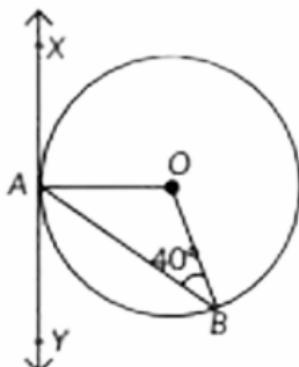
$$\begin{aligned}\angle C &= 180^\circ - \angle A - \angle B \\ &= 180^\circ - 30^\circ - 90^\circ \\ &= 150^\circ - 90^\circ \\ &= 60^\circ\end{aligned}$$

[CBSE 2021, 2013]

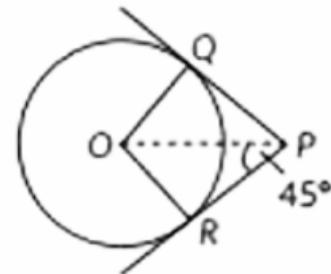
11

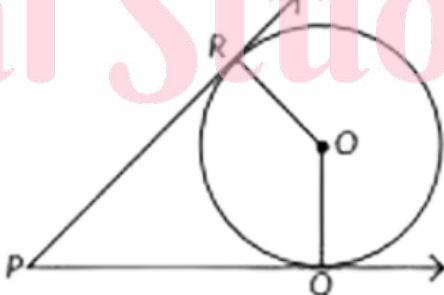
In Fig. XAY is a tangent to the circle centered at O. If $\angle ABO = 40^\circ$, then find $m\angle BAY$ and $m\angle AOB$.

2



	<p>Given, $\angle ABO = 40^\circ$ $\angle XAO = 90^\circ$... (Angle between radius and tangent) $OA = OB$... (Radii of same circle) $\Rightarrow \angle OAB = \angle OBA$ $\therefore \angle OAB = 40^\circ$</p> <p>Now, applying the linear pair of angles property, we get</p> $\begin{aligned}\angle BAY + \angle OAB + \angle XAO &= 180^\circ \\ \Rightarrow \angle BAY + 40^\circ + 90^\circ &= 180^\circ \\ \Rightarrow \angle BAY + 130^\circ &= 180^\circ \\ \Rightarrow \angle BAY &= 180^\circ - 130^\circ \\ \Rightarrow \angle BAY &= 50^\circ\end{aligned}$ <p>Now, In $\triangle AOB$,</p> $\begin{aligned}\angle AOB + \angle OAB + \angle OBA &= 180^\circ \\ \text{or, } \angle AOB + 40^\circ + 40^\circ &= 180^\circ\end{aligned}$ <p style="text-align: right;">[CBSE 2021]</p>	
12	<p>In Figure, PQ and PR are tangents to the circle centered at O. If $\angle OPR = 45^\circ$, then prove that ORPQ is a square.</p>	2



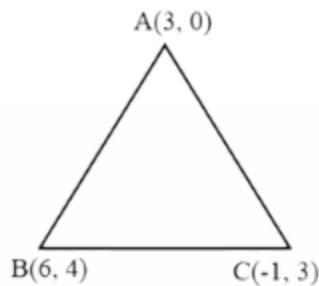
	<p>It is given that $\angle QPR = 90^\circ$</p> <p>We know that the lengths of the tangents drawn from the outer point to the circle are equal.</p> <p>$PQ = PR \dots (1)$</p> <p>The radius is Perpendicular to the tangent line at the point of contact.</p> <p>$\therefore \angle PQO = 90^\circ$ and $\angle ORP = 90^\circ$</p> <p>In quadrilateral OQPR:</p> $\angle QPR + \angle PQO + \angle QOR + \angle ORP = 360^\circ$ $\Rightarrow 90^\circ + 90^\circ + \angle QOR + 90^\circ = 360^\circ$ $\Rightarrow \angle QOR = 360^\circ - 270^\circ = 90^\circ$ $\therefore \angle QPR = \angle PQO = \angle QOR = \angle ORP = 90^\circ$ <p>It can be concluded that $PQOR$ is a square.</p>	
13	<p>In Figure. O is the center of the circle. PQ and PR are tangent segments. Show that the quadrilateral PQOR is cyclic.</p> 	2

	<p>PR and PO are the two tangents to the circle from an external point P.</p> <p>We know that the radius of a circle is perpendicular to the tangent at the point of contact.</p> <p>So, OR \perp PR and OQ \perp PQ</p> <p>$\angle ORP = \angle OQP = 90^\circ$</p> <p>We know that the sum of all interior angles in a quadrilateral is always equal to 360°</p> <p>Considering quadrilateral POOR,</p> <p>$\angle OQP + \angle QOR + \angle ORP + \angle RPQ = 360^\circ$</p> <p>$90^\circ + \angle QOR + 90^\circ + \angle RPQ = 360^\circ$</p> <p>$180^\circ + \angle QOR + \angle RPQ = 360^\circ$</p> <p>$\angle QOR + \angle RPQ = 360^\circ - 180^\circ$</p> <p>So, $\angle O + \angle P = 180^\circ$</p> <p>Here opposite angles are supplementary.</p> <p>Therefore, POOR is a cyclic quadrilateral.</p> <p style="text-align: right;">[CBSE 2016, 2014]</p>	
14	<p>A train travels at a certain average speed for a distance 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than the original speed. If it takes 3 hours to complete the total journey, what is its original average speed?</p> <p>Let x represent the train's initial average speed. The time required by the train to travel 63 km at the original speed is equal to $63/x$.</p> <p>Time taken by train to cover 72 km with increased speed = $72/(x + 6)$</p> <p>According to the question it is given:</p> $63/x + 72/(x + 6) = 3$ $[63(x + 6) + 72x]/x(x + 6) = 3$ $[63x + 378 + 72x]/(x^2 + 6x) = 3$	2

	<p>In simplification we get the:</p> $378 + 135x = 3x^2 + 18x$ $3x^2 - 117x - 378 = 0$ $x^2 - 39x - 126 = 0$ $x^2 - 42x + 3x - 126 = 0$ <p>Taking common we get:</p> $x(x - 42) + 3(x - 42) = 0$ $(x + 3)(x - 42) = 0$ <p>We get:</p> $x = 42, -3$ <p>Since speed cannot be zero, the initial average speed is 42 km per hour.</p>	
15	<p>In figure, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = BC + AD$</p>	3

	<p>We know that , tangents drawn from a point outside the circle are equal in length.</p> <p>So,</p> <p>$AS = AP$</p> <p>$BQ = BP$</p> <p>$CQ = CR$</p> <p>$DS = DR$</p> <p>On adding the above equations we get,</p> <p>$AS + BQ + CQ + DS = AP + BP + CR + DR$</p> <p>$(AS + DS) + (BQ + CQ) = (AP + BP) + (CR + DR)$</p> <p>$\Rightarrow AD + BC = AB + CD.$</p>	
16	<p>If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$.</p> <p style="text-align: right;">[CBSE 2017]</p>	2
17	<p>Find the ratio in which the y-axis divides the line segment joining the points $(6, -4)$ and $(-2, -7)$. Also, find the point of intersection.</p> <p>Given: Line segment joining the points $(6, -4)$ and $(-2, -7)$</p> <p>To find:</p> <ol style="list-style-type: none"> 1. The ratio in which the y-axis divides the line segment joining the points. 2. The coordinates of point of intersection. <p>Formula used: $\text{slope of line} = \frac{y_2 - y_1}{x_2 - x_1}$</p> <p>Explanation:</p> <p>1. The ratio of line segment.</p> <p>I have attached a graph below</p> <p>The line can be divided in at $y = 0$</p> <p>So length of first segment = $0 - (-2) = 2$ unit</p> <p>Length of second segment = $6 - 0 = 6$ unit</p> <p>Ratio of first segment and second segment = $2 : 6 = 1 : 3$</p> <p>Hence, the ratio is 1 : 3.</p>	3

	<p>2. The coordinates of point of intersection.</p> <p>Slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$</p> <p>From given line,</p> $(6, -4) = (x_1, y_1)$ $(-2, -7) = (x_2, y_2)$ $\text{slope of line} = \frac{-7 - (-4)}{-2 - 6} = 0.375$ <p>Slope of line can never be changed.</p> <p>So, now we consider</p> $(x_1, y_1) = (6, -4)$ $(x_2, y_2) = (0, y)$ $\text{Slope of line} = \frac{y_2 - y_1}{x_2 - x_1}$ $0.375 = \frac{y - (-4)}{0 - 6}$ $0.375 = \frac{y + 4}{-6}$ $y + 4 = 0.375 \times (-6)$ $y + 4 = -2.25$ $y = -2.25 - 4$ $y = -6.25$ <p>Therefore, the co-ordinates of point of intersection is $y = -6.25$.</p> <p style="text-align: right;">[CBSE 2020]</p>	
18	Prove that the points (3,0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.	2



$$\text{Length of } AB = \sqrt{(6-3)^2 + (4-0)^2}$$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

$$\text{Length of } BC = \sqrt{(-1-6)^2 + (3-4)^2}$$

$$= \sqrt{(-7)^2 + (-1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} \text{ units.}$$

$$\text{And Length of } AC = \sqrt{(-1-3)^2 + (3-0)^2}$$

$$= \sqrt{(-4)^2 + (3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

$$\therefore AB = AC$$

$$\text{And } (AB)^2 + (AC)^2 = (BC)^2$$

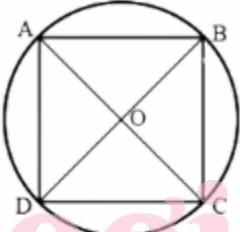
Hence, $\triangle ABC$ is a isosceles, right angled triangle.

[CBSE 2017, 2020]

19

Find the value(s) of x , if the distance between the points $A(0, 0)$ and $B(x, 4)$ is 5 units. (2019)

1

	<p>The distance between the two points A(0, 0) and B(x, -4) is 5 units.</p> <p>We know that, The distance between the two points A(x_1, y_1) and B(x_2, y_2)</p> $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\Rightarrow \sqrt{(x - 0)^2 + (-4 - 0)^2} = 5$ $\Rightarrow \sqrt{x^2 + 16} = 5$ $\Rightarrow x^2 + 16 = 5^2 = 25$ $\Rightarrow x^2 = 25 - 16$ $\Rightarrow x^2 = 9$ $\Rightarrow x = \pm 3$ <p>Hence, the value(s) of x are 3 and - 3.</p>	
20	<p>If A(4, 3), B(-1, y) and C(3, 4) are the vertices of a right triangle ABC, right-angled at A, then find the value of y.</p>  <p>Given the triangle ABC, right angled at A.</p> <p>Now, $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</p> $AB = \sqrt{(-1 - 4)^2 + (y - 3)^2}$ $AB = \sqrt{(-5)^2 + (y - 3)^2}$ $AB = \sqrt{25 + (y - 3)^2}$ $AB = \sqrt{25 + y^2 + 9 - 6y}$ $AB = \sqrt{34 + y^2 - 6y}$	2

$$BC = \sqrt{(3 - (-1))^2 + (4 - y)^2}$$

$$BC = \sqrt{(4)^2 + (4 - y)^2}$$

$$BC = \sqrt{16 + 16 + y^2 - 8y}$$

$$BC = \sqrt{32 + y^2 - 8y}$$

$$AC = \sqrt{(3 - 4)^2 + (4 - 3)^2}$$

$$AC = \sqrt{(-1)^2 + (1)^2}$$

$$AC = \sqrt{1+1}$$

$$AC = \sqrt{2} \text{ units}$$

Given, $\triangle ABC$ is a right angled triangle, right angled at A

So, by Pythagoras theorem $BC^2 = AC^2 + AB^2$

$$(\sqrt{32 + y^2 - 8y})^2 = (\sqrt{2})^2 + (\sqrt{32 + y^2 - 6y})^2$$

$$32 + y^2 - 8y = 2 + 34 + y^2 - 6y$$

$$-2y = 4$$

$$y = -2$$

Hence, the value of y is -2.

[CBSE Term II, 2021-22]

SUBJECT: MATHS

CLASS: X

CHAPTER: Trigonometry & Statistics

No. of PYQs:20

SI No	QUESTIONS	MARK
1	<p>If $2 \tan A = 3$, then the value of $(4\sin A + 3\cos A) / (4\sin A - 3\cos A)$ is</p> <p>Solution- Given $2 \tan A = 3$ $\tan A = 3/2$ $\sin A = 3/\sqrt{13}$ $\cos A = 2/\sqrt{13}$</p> <p>So $(4 \sin A + 3 \cos A) / (4 \sin A - 3 \cos A)$</p> $(4 \times \frac{3}{\sqrt{13}} + 3 \times \frac{2}{\sqrt{13}}) / (4 \times \frac{3}{\sqrt{13}} - 3 \times \frac{2}{\sqrt{13}}) = 3$ <p style="text-align: right;">[CBSE 2023]</p>	1
2	<p>If $\tan \theta + \cot \theta = \frac{4\sqrt{3}}{3}$ then find the value of $\tan^2 \theta + \cot^2 \theta$?</p> <p>Solution- $\tan \theta + \cot \theta = \frac{4\sqrt{3}}{3}$ Squaring both the side, $\tan^2 \theta + \cot^2 \theta + 2 \tan \theta \cdot \cot \theta = \frac{16 \times 3}{9}$ $\Rightarrow \tan^2 \theta + \cot^2 \theta + 2(1) = \frac{48}{9}$ $\Rightarrow \tan^2 \theta + \cot^2 \theta = \frac{48}{9} - 2 = \frac{30}{9}$</p> <p style="text-align: right;">[CBSE 2021]</p>	1
3	<p>Given $15 \cot A = 8$, then find the values of $\sin A$ and $\sec A$.</p> <p>Solution- It is given that: $15 \cot A = 8$</p> $\Rightarrow \cot A = \frac{8}{15}$ $\Rightarrow \cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{8}{15}$ Base=8k and Perpendicular= 15k Hypotenuse= $\sqrt{\text{base}^2 + \text{perpendicular}^2}$ $\Rightarrow H = \sqrt{(8k)^2 + (15k)^2} = 17k$ $\sin A = \frac{15k}{17k} = \frac{15}{17}$ $\sec A = \frac{17k}{8k} = \frac{17}{8}$ <p style="text-align: right;">[CBSE2020C]</p>	2

4	<p>Given $\sin A = 3/5$ find the other trigonometric ratios of the angle A.</p> <p>Solution- $\sin A = \frac{3}{5}$</p> <p>$\Rightarrow \sin A = \frac{p}{h}$, so $p = 3k$ & $h = 5k$</p> <p>base = $\sqrt{hyp^2 - perpendicular^2}$</p> <p>$\Rightarrow b = \sqrt{(5k)^2 - (3k)^2} = 4k$</p> <p>$\cos A = b/h = 4k/5k = \frac{4}{5}$</p> <p>$\tan A = p/b = 3k/4k = \frac{3}{4}$</p> <p>$\cosec A = 1/\sin A = 5/3$</p> <p>$\sec A = 1/\cos A = 5/4$</p> <p>$\cot A = 1/\tan A = 4/3$</p>	3
[CBSE 2016]		
5	<p>$[\frac{5}{8} \sec^2 60^\circ - \tan^2 60^\circ + \cos^2 45^\circ]$ is equal to</p>	1
Solution-	$\sec 60^\circ = \frac{2}{\sqrt{3}}$, $\tan 60 = \sqrt{3}$, $\cos 45 = \frac{1}{\sqrt{2}}$	
$\frac{5}{8} \times \frac{4}{3} - 3 + \frac{1}{2} = \frac{5}{6} - \frac{5}{2} = -\frac{10}{6} = -\frac{5}{3}$	[CBSE 2023, 2015]	
6	<p>The value of θ for which $2 \sin 2\theta = 1$, is</p> <p>Solution- $2 \sin 2\theta = 1$</p> <p>$\Rightarrow \sin 2\theta = \frac{1}{2}$</p> <p>$\Rightarrow \sin 2\theta = \sin 30^\circ$</p> <p>$\Rightarrow \theta = 15^\circ$</p>	1
[CBSE 2021, Sample Paper 2019]		
7	<p>If $\sin x + \cos y = 1$; $x = 30^\circ$ and y is an acute angle, find the value of y.</p> <p>Solution- $\sin x + \cos y = 1$</p> <p>$\Rightarrow \sin 30 + \cos y = 1$</p> <p>$\Rightarrow \frac{1}{2} + \cos y = 1$</p> <p>$\Rightarrow \cos y = \frac{1}{2}$</p> <p>$\Rightarrow \cos y = \cos 60$</p> <p>$\Rightarrow y = 60^\circ$</p>	1
[CBSE 2019]		
8	<p>Evaluate $2\sec^2\theta + 3\cosec^2\theta - 2\sin\theta\cos\theta$ if $\theta = 45^\circ$?</p> <p>Solution- $2\sec^2\theta + 3\cosec^2\theta - 2\sin\theta\cos\theta$</p> <p>Here, $\sec 45 = \cosec 45 = \sqrt{2}$, $\sin 45 = \frac{1}{\sqrt{2}} = \cos 45$</p>	2

	$= 2 \times (\sqrt{2})^2 + 3(\sqrt{2})^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$ $= 4 + 6 - 1 = 9$	
9	<p style="text-align: right;">[CBSE 2015, 2023]</p> <p>Prove that $(1 + \sec\theta - \tan\theta)/(1 + \sec\theta + \tan\theta) = (1 - \sin\theta)/\cos\theta$</p> <p>Solution - We have to prove that $(1 + \sec\theta - \tan\theta)/(1 + \sec\theta + \tan\theta) = (1 - \sin\theta)/\cos\theta$</p> <p>Considering LHS,</p> <p>LHS : $(1 + \sec\theta - \tan\theta)/(1 + \sec\theta + \tan\theta)$</p> <p>By using, $\sec^2 A - \tan^2 A = 1$</p> $(1 + \sec\theta - \tan\theta) = \sec\theta - \tan\theta + (\sec^2\theta - \tan^2\theta)$ $= (\sec\theta - \tan\theta) + [(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)]$ <p>Taking out common term,</p> $= (\sec\theta - \tan\theta)[1 + \sec\theta + \tan\theta]$ <p>Now, $(1 + \sec\theta - \tan\theta)/(1 + \sec\theta + \tan\theta) = (\sec\theta - \tan\theta)[1 + \sec\theta + \tan\theta] / (1 + \sec\theta + \tan\theta)$</p> $= (\sec\theta - \tan\theta)$ <p>We know that $\sec A = 1/\cos A$ and $\tan A = \sin A/\cos A$</p> <p>So, $(\sec\theta - \tan\theta) = (1/\cos\theta - \sin\theta/\cos\theta)$</p> $= (1 - \sin\theta)/\cos\theta$ <p>= RHS</p> <p>LHS = RHS</p>	3

	<p>Therefore, $(1 + \sec\theta - \tan\theta)/(1 + \sec\theta + \tan\theta) = (1 - \sin\theta)/\cos\theta$</p> <p style="text-align: right;">[CBSE 2020]</p>	
10	<p>Prove that $(\cot A + \sec B)^2 - (\tan B - \cosec A)^2 = 2(\cot A \cdot \sec B + \tan B \cdot \cosec A)$</p> <p>Solution- $(\tan A + \cosec B)^2 - (\cot B - \sec A)^2$ $= (\tan^2 A + 2\tan A \cosec B + \cosec^2 B) - (\cot^2 B - 2\cot B \sec A + \sec^2 A)$ $= \tan^2 A + \cosec^2 B + 2\tan A \cosec B - \cot^2 B - \sec^2 A + 2\cot B \sec A$</p> <p>Substituting $(\tan^2 A = \sec^2 A - 1)$ and $(\cosec^2 B = 1 + \cot^2 B)$ in the above step:</p> $(\sec^2 A - 1) + (1 + \cot^2 B) + 2\tan A \cosec B - \cot^2 B - \sec^2 A + 2\cot B \sec A$ $= 2\tan A \cosec B + 2\cot B \sec A$ $= 2(\tan A \cosec B + \cot B \sec A)$ <p>RHS,</p> $2\tan A \cot B (\cosec A + \sec B)$ $= 2\tan A \cot B ((1 / \sin A) + (1 / \cos B))$ $= 2(\tan A / \sin A) \cot B + 2\tan A (\cot B / \cos B)$ $= 2(\sin A / (\cos A \sin B)) \cot B + 2\tan A (\cos B / (\sin B \cos B))$ $= 2(1 / \cos A) \cot B + 2 \tan A (1 / \sin B)$ $= 2\sec A \cot B + 2\tan A \cosec B$ $= 2(\sec A \cot B + \tan A \cosec B)$ <p style="text-align: right;">[CBSE 2013, 2017]</p>	4
11	<p>If the mean of the first n natural numbers is 15, then find n.</p> <p>unique country?</p> <p>Solution- $\frac{1+2+3+\dots}{n} = 15$</p> $\Rightarrow \frac{\frac{n(n+1)}{2}}{n} = 15$ $\Rightarrow n + 1 = 15 \times 2$ $\Rightarrow n = 29$ <p style="text-align: right;">[CBSE 2015, Sample Paper 2020]</p>	1
12	<p>If the mean of 5 observations $x, x + 2, x + 4, x + 6$ and $x + 8$ is 11, then find the value of x.</p> <p>Solution -</p> $\frac{x+x+2+x+4+x+6+x+8}{5} = 11$ $\Rightarrow 5x + 20 = 55$ $\Rightarrow x = 7$	1

	[CBSE 2015]													
13	<p>The mean weight of 150 students in a class is 60 kg. The mean weight of boys is 70 kg while that of girls is 55 kg. Find the number of boys and girls in the class.</p> <p>Solution - Let the number of boys be denoted by b and number of girls be denoted by g</p> <p>We know that, $\text{Sum of all observations} = \text{Mean of observations} \times \text{No. of obs.}$ Therefore,</p> <p style="background-color: #e0f2e0; padding: 10px;"> $\text{Sum of weight of boys} = 70b$ $\text{Sum of weight of girls} = 55g$ $\text{Total weight of all students} = 60 \times 150$ Therefore, $(bx70) + (g \times 55) = (b + g)60$ $70b + 55g = 150 \times 60$ $70b + 55g = 9000 \dots (i)$ Also, $b + g = 150$ or $b = 150 - g \dots (ii)$ Solving (1) and (2), we get, $70(150g) + 55g = 9000$ $10500 - 70g + 55g = 9000$ $1500 = 15g \therefore g = 100$ Hence, $b = 150 - 100 = 50$ </p>	3												
14	<p>If the mean of the following frequency distribution is 10.8, then find the value of p:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Class</td> <td>0-4</td> <td>4-8</td> <td>8-12</td> <td>12-16</td> <td>16-20</td> </tr> <tr> <td>f</td> <td>3</td> <td>p</td> <td>5</td> <td>8</td> <td>2</td> </tr> </table> <p>Solution -</p> <p>\Rightarrow Here, $\frac{3 + p + 5 + 8 + 2}{5} = 10.8$ $18 + p = 54$ Thus, $p = 36.$</p>	Class	0-4	4-8	8-12	12-16	16-20	f	3	p	5	8	2	2
Class	0-4	4-8	8-12	12-16	16-20									
f	3	p	5	8	2									
15	<p>The arithmetic mean of the following frequency distribution is 53. Find the value of k.</p>	3												

Class	0-20	20-40	40-60	60-80	80-100
f	12	15	32	k	13

Solution-

$$\Rightarrow \text{Here, } \frac{3+p+5+8+2}{5} = 10.8$$

$$18 + p = 54$$

$$\text{Thus, } p = 36$$

[CBSE 2021-22]

16

Write the empirical relationship between the three measures of central tendency.

Solution - 2 Mean = 3 Median – Mode

[CBSE 2021, 2017]

1

17

Find the mean of the data, using an empirical formula, when it is given that mode = 50.5 and median = 45.5.

Solution - 2 Mean = 3M-Z

$$\Rightarrow 2\bar{x} = 3 \times 45.5 - 50.5$$

$$\Rightarrow \bar{x} = 86$$

$$\Rightarrow \bar{x} = 43$$

[CBSE 2017]

1

18

For the following frequency distribution, find the median:

Class	1400-1550	1550-1700	1700-1850	1850-2000
f	6	13	25	10

Solution:-

C-I	f	c.f.
1400-1550	6	6
1550-1700	13	19
1700-1850	25	44
1850-2000	10	54

$$N/2 = 54/2 = 27$$

The cumulative frequency greater than or equal to 27 is 44
the class interval 1700-1850 so median class is 1700-1850

	[CBSE 2021, 2014]																																														
19	<p>The median of the following data is 16. Find the missing frequencies a and b, if the total of the frequencies is 70.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>C-I</td><td>0-5</td><td>5-10</td><td>10-15</td><td>15-20</td><td>20-25</td><td>25-30</td><td>30-35</td><td>35-40</td></tr> <tr> <td>f</td><td>12</td><td>a</td><td>12</td><td>15</td><td>b</td><td>6</td><td>6</td><td>4</td></tr> </table> <p>Solution-</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>C-I</th><th>f</th><th>c.f.</th></tr> </thead> <tbody> <tr> <td>0-5</td><td>12</td><td>12</td></tr> <tr> <td>5-10</td><td>a</td><td>12+a</td></tr> <tr> <td>10-15</td><td>12</td><td>24+a</td></tr> <tr> <td>15-20</td><td>15</td><td>39+a</td></tr> <tr> <td>20-25</td><td>b</td><td>39+a+b</td></tr> <tr> <td>25-30</td><td>6</td><td>45+a+b</td></tr> <tr> <td>30-35</td><td>6</td><td>51+a+b</td></tr> <tr> <td>35-40</td><td>4</td><td>55+a+b</td></tr> </tbody> </table> <p>$N=70 = a+b+55$ $So, a+b= 15$ Median is 16, which lies in 15-20. So the median class is 15-20 . Therefore, $I = 15$, $h= 5$, $N= 70$, $f = 15$, and $cf = 24+a$</p> $M = I + \frac{\frac{N}{2} - c}{f} \times h$ $\Rightarrow 16 = 15 + \frac{35-24-a}{15} \times 5$ <p>After solving , $a=8$ so $b=7$</p> <p style="text-align: right;">[CBSE 2015, 2020]</p>	C-I	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	f	12	a	12	15	b	6	6	4	C-I	f	c.f.	0-5	12	12	5-10	a	12+a	10-15	12	24+a	15-20	15	39+a	20-25	b	39+a+b	25-30	6	45+a+b	30-35	6	51+a+b	35-40	4	55+a+b	3
C-I	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40																																							
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20	<p>If the difference of Mode and Median of a data is 24, then the difference of median and mean is</p> <p>Solution- Mode - Median = 24</p> <p>\therefore</p> <p>Mode = 24 + median</p> <p>But mode = 3 median - 2 mean</p> <p>\therefore</p> <p>3 median - 2 mean = 24 + median</p>	1																																													

	<p>3 median - median - 2 mean = 24 ⇒ 2 median - 2 mean = 24 ⇒ Median - Mean = 12</p>	
	<p>[CBSE 2022]</p>	

Logical Study